Supplemental Material

S.1 Proof of Lemma 3

The first inequality is easy to obtain by

$$\begin{aligned} P\{|(\hat{\gamma}_1-\hat{\gamma}_2)-(\gamma_1-\gamma_2)|>\epsilon\} &\leq P\{|\hat{\gamma}_1-\gamma_1|>\epsilon/2\}+P\{|\hat{\gamma}_2-\gamma_2|>\epsilon/2\} \\ &\leq b_3\exp(-n^{\nu}/b_3), \end{aligned}$$

where $b_3=b_1+b_2$. To study $\hat{\gamma}_1\hat{\gamma}_2$, we first show that $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are bounded in probability. Denote $M_1=\max\{|\gamma_1|+1/2,|\gamma_2|+1/2\}$, then

$$\begin{aligned} P\{|\hat{\gamma}_1|>M_1\} &\leq P\{|\hat{\gamma}_1-\gamma_1|+|\gamma_1|>M_1\} \\ &\leq P\{|\hat{\gamma}_1-\gamma_1|>1/2\}\leq b_1\exp(-n^{\nu}/b_1). \end{aligned} \tag{S.1}$$

Similarly, $P\{|\hat{\gamma}_2|>M_1\}\leq b_2\exp(-n^{\nu}/b_2)$. Then for any $0<\epsilon<1$,

$$\begin{aligned} & P\{|\hat{\gamma}_1\hat{\gamma}_2-\gamma_1\gamma_2|>\epsilon\} \\ = & P\{|\hat{\gamma}_1\hat{\gamma}_2-\hat{\gamma}_1\gamma_2+\hat{\gamma}_1\gamma_2-\gamma_1\gamma_2|>\epsilon\} \\ \leq & P\{|\hat{\gamma}_1|\cdot|\hat{\gamma}_2-\gamma_2|>\epsilon/2\}+P\{|\gamma_2|\cdot|\hat{\gamma}_1-\gamma_1|>\epsilon/2\} \\ \leq & P\{|\hat{\gamma}_1|\cdot|\hat{\gamma}_2-\gamma_2|>\epsilon/2,|\hat{\gamma}_1|\leq M_1\}+P\{|\hat{\gamma}_1|>M_1\}+P\{|\hat{\gamma}_1-\gamma_1|>\epsilon/(2M_1)\} \\ \leq & P\{|\hat{\gamma}_2-\gamma_2|>\epsilon/(2M_1)\}+P\{|\hat{\gamma}_1|>M_1\}+P\{|\hat{\gamma}_1-\gamma_1|>\epsilon/(2M_1)\} \end{aligned}$$

Thus by (B.3) and (S.1), $P\{|\hat{\gamma}_1\hat{\gamma}_2-\gamma_1\gamma_2|>\epsilon\}\leq b_4\exp(-n^{\nu}/b_4)$, where $b_4=2b_1+b_2$.

Now consider $\hat{\gamma}_1/\hat{\gamma}_2$ when $\gamma_2\neq 0$. Note that $\hat{\gamma}_2$ is bounded away from 0 with probability tending to 1. This is because $P(|\hat{\gamma}_2|<|\gamma_2|/2)=P(|\gamma_2-(\gamma_2-\hat{\gamma}_2)|<$

$|\gamma_2|/2) \le P(|\hat{\gamma}_2 - \gamma_2| > |\gamma_2|/2) \le b_2 \exp(-n^\nu/b_2)$, which tends to 0. Then

$$\begin{aligned}
& P\left\{\left|\frac{\hat{\gamma}_1}{\hat{\gamma}_2} - \frac{\gamma_1}{\gamma_2}\right| > \epsilon\right\} \\
\le\ & P\left\{\left|\frac{\hat{\gamma}_1}{\hat{\gamma}_2} - \frac{\gamma_1}{\hat{\gamma}_2}\right| > \epsilon/2\right\} + P\left\{\left|\frac{\gamma_1}{\hat{\gamma}_2} - \frac{\gamma_1}{\gamma_2}\right| > \epsilon/2\right\} \\
\le\ & P\left\{|\hat{\gamma}_1 - \gamma_1| > \frac{\epsilon|\hat{\gamma}_2|}{2}\right\} + P\left\{\left|\frac{\gamma_1}{\gamma_2\hat{\gamma}_2}\right| \cdot |\hat{\gamma}_2 - \gamma_2| > \epsilon/2\right\} \\
\le\ & P\left\{|\hat{\gamma}_1 - \gamma_1| > \frac{\epsilon|\hat{\gamma}_2|}{2}, |\hat{\gamma}_2| \ge \frac{|\gamma_2|}{2}\right\} + P\left\{|\hat{\gamma}_2 - \gamma_2| > \frac{\epsilon|\gamma_2\hat{\gamma}_2|}{2|\gamma_1|}, |\hat{\gamma}_2| \ge \frac{|\gamma_2|}{2}\right\} + 2P\left\{|\hat{\gamma}_2| < \frac{|\gamma_2|}{2}\right\} \\
\le\ & P\left\{|\hat{\gamma}_1 - \gamma_1| > \frac{\epsilon|\gamma_2|}{4}\right\} + P\left\{|\hat{\gamma}_2 - \gamma_2| > \frac{\epsilon\gamma_2^2}{4|\gamma_1|}\right\} + 2P\left\{|\hat{\gamma}_2| < \frac{|\gamma_2|}{2}\right\} \\
\le\ & b_1 \exp(-n^\nu/b_1) + b_2 \exp(-n^\nu/b_2) + 2b_2 \exp(-n^\nu/b_2).
\end{aligned}$$

Therefore, $P\left\{|\hat{\gamma}_1/\hat{\gamma}_2 - \gamma_1/\gamma_2| > \epsilon\right\} \le b_5 \exp(-n^\nu/b_5)$, where $b_5 = b_1 + 3b_2$.

If further assume $\gamma_2 > 0$, using the same technique as above,

$$\begin{aligned}
& P\left\{|\sqrt{\hat{\gamma}_2} - \sqrt{\gamma_2}| > \epsilon\right\} \\
\le\ & P\left\{\frac{|\hat{\gamma}_2 - \gamma_2|}{\sqrt{\hat{\gamma}_2} + \sqrt{\gamma_2}} > \epsilon, |\hat{\gamma}_2| \ge \frac{\gamma_2}{2}\right\} + P\left\{|\hat{\gamma}_2| < \frac{\gamma_2}{2}\right\} \\
\le\ & P\left\{|\hat{\gamma}_2 - \gamma_2| > \epsilon\sqrt{\gamma_2}(1 + \frac{1}{\sqrt{2}})\right\} + P\left\{|\hat{\gamma}_2| < \frac{\gamma_2}{2}\right\}.
\end{aligned}$$

Thus $P\left\{|\sqrt{\hat{\gamma}_2} - \sqrt{\gamma_2}| > \epsilon\right\} \le b_6 \exp(-n^\nu/b_6)$, where $b_6 = 2b_2$.

At last, since $\hat{\gamma}_2$ is consistent with γ_2, we can apply Taylor's expansion to $\log \hat{\gamma}_2$, i.e. $\log \hat{\gamma}_2 = \log \gamma_2 + (\hat{\gamma}_2 - \gamma_2)/\gamma_2 + o_p(\hat{\gamma}_2 - \gamma_2)$. Thus for large n,

$$P\left\{|\log \hat{\gamma}_2 - \log \gamma_2| > \epsilon\right\} \le P\left\{\frac{2}{\gamma_2}|\hat{\gamma}_2 - \gamma_2| > \epsilon\right\} \le P\left\{|\hat{\gamma}_2 - \gamma_2| > \delta'''\right\},$$

where $\delta''' = \min\{\epsilon, \epsilon\gamma_2/2\}$. Therefore, $P\left\{|\log \hat{\gamma}_2 - \log \gamma_2| > \epsilon\right\} \le b_2 \exp(-n^\nu/b_2)$.

S.2 Additional Simulation Results

This section provides additional simulation results. Table S1 depicts the

Table S1: Computational time (in minutes) of 1000 simulations when $p = 2000$

ρ	SCAD	LASSO	PC-simple	TPC
0	344.31	31.37	349.84	27.90
0.3	341.52	14.14	245.88	28.01
0.8	288.43	214.18	218.22	29.42

computing time of 1000 simulation with $p = 2000$ when data were generated from an elliptical distribution. Table S2 depicts the results from the normal linear model in the simulation examples. Table S3 reports the simulation results when data were generated a population in which x's with even subscripts were generated in the same fashion as that for elliptical distribution, and x's with odd subscripts take discrete values 0, 1 and 2 with probabilities 0.25, 0.5 and 0.25, respectively. In this simulation study, we take $\rho = 0.3$ low correlation and $\rho = 0.8$ for high correlation.

Table S2: Simulation Results for Example 1: Normal Distribution

p	ρ	Method	MedME(Devi)	TPN	TFN	UF	CF	OF
200	0	SCAD	0.013 (0.006)	3.00	1.04	0.00	0.66	0.34
		LASSO	8.936 (0.148)	3.00	16.91	0.00	0.01	0.99
		PC-simple	0.012 (0.006)	3.00	0.03	0.00	0.97	0.03
		TPC	0.012 (0.006)	3.00	0.03	0.00	0.97	0.03
200	0.3	SCAD	0.014 (0.006)	3.00	0.86	0.00	0.73	0.27
		LASSO	11.105 (0.151)	3.00	15.60	0.00	0.02	0.98
		PC-simple	0.011 (0.006)	3.00	0.00	0.00	1.00	0.00
		TPC	0.011 (0.006)	3.00	0.01	0.00	0.99	0.01
200	0.8	SCAD	0.010 (0.006)	3.00	0.67	0.00	0.72	0.28
		LASSO	20.731 (0.069)	3.00	9.52	0.00	0.03	0.97
		PC-simple	0.009 (0.006)	2.92	0.10	0.08	0.92	0.00
		TPC	0.009 (0.006)	2.92	0.10	0.08	0.92	0.00
500	0	SCAD	0.013 (0.008)	3.00	1.26	0.00	0.77	0.23
		LASSO	9.046 (0.121)	3.00	20.75	0.00	0.02	0.98
		PC-simple	0.014 (0.008)	3.00	0.14	0.00	0.87	0.13
		TPC	0.014 (0.008)	3.00	0.15	0.00	0.86	0.14
500	0.3	SCAD	0.014 (0.007)	3.00	1.33	0.00	0.72	0.28
		LASSO	11.231 (0.101)	3.00	19.07	0.00	0.00	1.00
		PC-simple	0.013 (0.007)	3.00	0.07	0.00	0.93	0.07
		TPC	0.013 (0.008)	3.00	0.10	0.00	0.90	0.10
500	0.8	SCAD	0.011 (0.007)	3.00	0.92	0.00	0.71	0.29
		LASSO	20.777 (0.085)	3.00	11.74	0.00	0.02	0.98
		PC-simple	0.012 (0.008)	2.86	0.18	0.14	0.86	0.00
		TPC	0.012 (0.008)	2.87	0.16	0.13	0.87	0.00
2000	0	SCAD	0.013 (0.007)	3.00	2.25	0.00	0.66	0.34
		LASSO	9.080 (0.120)	3.00	31.21	0.00	0.00	1.00
		PC-simple	0.023 (0.017)	3.00	0.41	0.00	0.62	0.38
		TPC	0.022 (0.016)	3.00	0.41	0.00	0.62	0.38
2000	0.3	SCAD	0.010 (0.006)	3.00	1.65	0.00	0.69	0.31
		LASSO	11.277 (0.129)	3.00	26.97	0.00	0.00	1.00
		PC-simple	0.010 (0.006)	3.00	0.12	0.00	0.89	0.11
		TPC	0.010 (0.006)	3.00	0.13	0.00	0.88	0.12
2000	0.8	SCAD	0.011 (0.006)	3.00	1.32	0.00	0.66	0.34
		LASSO	20.828 (0.098)	3.00	15.50	0.00	0.03	0.97
		PC-simple	0.011 (0.007)	2.90	0.11	0.10	0.90	0.00
		TPC	0.011 (0.007)	2.90	0.11	0.10	0.90	0.00

$*$ The numbers in the parentheses are median absolute deviations over 1000 simulations.

Table S3: Simulation Results for Elliptical Distribution with half x's being discrete

p	ρ	Method	MedME(Devi)	TPN	FPN	UF	CF	OF
		SCAD	1.040 (0.892)	3.00	8.52	0.00	0.13	0.87
		LASSO	11.209 (0.220)	3.00	20.40	0.00	0.00	1.00
200	0.3	PC-simple	0.218 (0.054)	3.00	0.47	0.00	0.59	0.41
		TPC	0.187 (0.053)	3.00	0.14	0.00	0.87	0.13
		SCAD	0.114 (0.058)	3.00	4.71	0.00	0.22	0.78
		LASSO	20.619 (0.172)	3.00	16.53	0.00	0.00	1.00
200	0.8	PC-simple	0.090 (0.042)	2.97	0.33	0.03	0.72	0.25
		TPC	0.091 (0.038)	2.96	0.17	0.04	0.84	0.12
		SCAD	1.425 (1.160)	3.00	14.11	0.00	0.08	0.92
		LASSO	11.252 (0.221)	3.00	33.78	0.00	0.00	1.00
500	0.3	PC-simple	0.222 (0.063)	3.00	0.67	0.00	0.45	0.55
		TPC	0.198 (0.054)	2.99	0.32	0.01	0.69	0.30
		SCAD	0.119 (0.049)	3.00	8.48	0.00	0.13	0.87
		LASSO	20.659 (0.199)	3.00	24.43	0.00	0.00	1.00
500	0.8	PC-simple	0.099 (0.037)	2.99	0.37	0.01	0.69	0.30
		TPC	0.096 (0.033)	3.00	0.21	0.00	0.80	0.20
		SCAD	1.584 (0.893)	3.00	25.60	0.00	0.02	0.98
		LASSO	11.383 (0.219)	3.00	55.84	0.00	0.00	1.00
2000	0.3	PC-simple	0.295 (0.067)	3.00	1.51	0.00	0.11	0.89
		TPC	0.234 (0.066)	3.00	0.67	0.00	0.44	0.56
		SCAD	0.183 (0.079)	3.00	18.16	0.00	0.02	0.98
		LASSO	20.720 (0.168)	3.00	38.64	0.00	0.00	1.00
2000	0.8	PC-simple	0.127 (0.048)	2.98	0.74	0.02	0.41	0.57
		TPC	0.116 (0.037)	2.99	0.36	0.01	0.69	0.30